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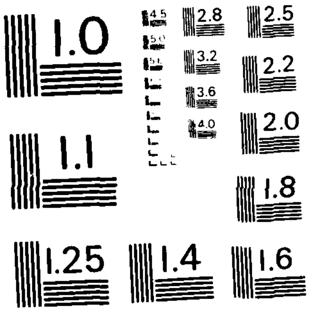

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NATIONAL BUREAU OF STANDARDS-1963-A

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1. REPORT NUMBER	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Nonlinear Inverse Heat Transfer Calculations in Gun Barrels	5. TYPE OF REPORT & PERIOD COVERED Interim Report 7-15-82 through 7-15-83
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Alfred S. Carasso	8. CONTRACT OR GRANT NUMBER(s) MIPR No. ARO 63-82
9. PERFORMING ORGANIZATION NAME AND ADDRESS Center for Applied Mathematics, National Bureau of Standards, Washington, DC 20234	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709	12. REPORT DATE July 15, 1983
	13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

JUL 29 1983

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inverse heat conduction; embedded thermocouple; nonlinear problem; temperature-dependent diffusion; ill-posed initial value problem; regularization in frequency domain; numerical experiments; 155mm cannon.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We consider the problem of determining the temperature history inside a gun barrel from embedded thermocouple measurements at some distance away from the inside wall. This inverse problem leads to an improperly posed initial value problem for a nonlinear system of partial differential equations, whenever the thermal properties are temperature dependent. We discuss a step-by-step marching algorithm for the numerical computation of such problems. The scheme is stabilized by appropriately filtering in the frequency domain at each step. We illustrate this technique with a numerical experiment on a nonlinear problem whose exact

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NONLINEAR INVERSE HEAT TRANSFER CALCULATIONS IN GUN BARRELS*

by

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ABSTRACT

We consider the problem of determining the temperature history inside a gun barrel from embedded thermocouple measurements at some distance away from the inside wall. This inverse problem leads to an improperly posed initial value problem for a nonlinear system of partial differential equations, whenever the thermal properties are temperature dependent. We discuss a step-by-step marching algorithm for the numerical computation of such problems. The scheme is stabilized by appropriately filtering in the frequency domain at each step. We illustrate this technique with a numerical experiment on a nonlinear problem whose exact solution is known. The basic ideas are applicable to other unstable evolution equations.

I. Introduction

This report summarizes the results of an important computational experiment on a nonlinear inverse heat conduction problem whose exact solution is known. We consider the problem of determining the temperature history at the inside wall of a gun barrel, from embedded thermocouple measurements at various points in the annular metallic region between the inner and outer radii of the cannon. As the shell is fired, a continuous trace is recorded at each thermocouple, providing temperature as a function of time at the corresponding fixed spatial location.

* Research sponsored by the U.S. Army Research Office under MIPR No. ARO 63-82

The present study centers around a novel computational technique designed especially for coping with the nonlinear case of temperature dependent thermal properties. It is a sequel to [1] where the linear quarter plane problem with constant coefficients, was thoroughly analyzed. As was shown there, in that case, the inverse problem can be formulated either as a Volterra integral equation of the first kind, or equivalently, as an initial value problem for the one dimensional heat equation run sideways. Either formulation leads to an improperly posed problem in which the solution, when it exists, depends discontinuously on the data.

The inverse problem can be regularized in the L^2 norm by placing an a-priori bound M on the norm of the unknown temperature history, $f(t)$, at the inside wall $x = 0$; at the same time, the measured noisy temperature data $g_m(t)$, at the location $x = \ell > 0$, is regarded as differing by at most ϵ in the L^2 norm from unknown smooth exact data $g(t)$, for which a solution exists. It is assumed that ϵ and M are known and compatible. As shown in [1, equations (2.20), (2.21)] this leads to explicit formulae for the temperature and gradient histories at each fixed x, $0 < x < \ell$. Also, error estimates are obtained for the regularized solutions implying Holder continuity with respect to the data, for each fixed positive x. These estimates degenerate at the wall, [1, Theorem 1].

The regularization procedure can be interpreted as solving the initial value problem for the sideways heat equation with appropriately modified initial data. An explicit finite difference

scheme consistent with that problem is shown to be unconditionally convergent, when used with the filtered initial data, [1, Theorem 3]. This step-by-step marching scheme in the x-variable is the basis for our approach to the nonlinear case of a temperature-dependent diffusion coefficient. We regularize the calculation at each step by filtering in the frequency domain, using FFT algorithms; we then return to the physical variables for the calculation of the next step. The filtering function used at each step is that determined by the related constant coefficient problem. This algorithm is outlined in [1, Section 7].

In order to test the robustness of this procedure, an example was manufactured with a known exact solution. This is a fictitious mathematical problem, artificially created so as to have a solution which simulates conditions presumed to exist in a 155mm cannon. The relevant parameters were made available to us by Dr. A. K. Celmins, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Grounds, Maryland. The "exact" solution was constructed numerically by solving a well posed direct problem as explained below.

2. The Direct Problem

Consider the initial boundary value problem

$$(2.1) \qquad \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[a(u)\frac{\partial u}{\partial x}\right], \quad 0 < x < \ell, \ t > 0,$$

$$(2.2) \qquad u(0,t) = f(t), \qquad u(1,t) = h(t), \ t > 0,$$

$$(2.3) \qquad u(x,0) = 300^\circ\ K$$

where t is the time measured in milliseconds, x measured in millimeters represents distance away from the inside wall, and $u(x,t)$ is the temperature in degrees Kelvin. The heat conduction equation (2.1) is a simplification of the actual physical situation, in that first order terms arising from cylindrical symmetry have been neglected, as well as the variation of specific heat with temperature. Moreover, for gun steel at temperatures between 300° K and 1000° K, the conduction coefficient $a(u)$ in (2.1) is well approximated by a linear function of u,

$$(2.4) \qquad a(u) = \{1.299 - 1.144 \times 10^{-3} (u - 255)\} \times 10^{-2} \text{ mm}^2/\text{millisec}$$

We remark that the methodology to be discussed can easily accommodate the more exact differential equation, as well as more complicated dependencies of $a(u)$ on u. We shall refer to the quantity

$$(2.5) \qquad w(x,t) = -a(u) \frac{\partial u}{\partial x}$$

as the <u>temperature gradient</u>, by an abuse of terminology. It is measured in <u>mm° K/milliseconds</u>. In all Figures shown below dealing with plots of $w(x,t)$ as a function of t for some fixed x, the vertical axis bears the legend "temperature gradient."

The functions $f(t)$ and $h(t)$ in (2.2) represent, respectively, the temperature histories at the inside wall and at 1mm away from the wall. These mathematical functions are plotted in Figure 1; they are constructed so as to approximate observed temperature histories in gun barrels, [4].

The direct problem given by (2.1), (2.2), and (2.3) was solved numerically, using an adaptive partial differential equation software package, MOL1D, [3]. The numerical integration was carried out to a distance in time equal to 100 milliseconds. The temperature $u(x,t)$ and gradient $w(x,t)$ were evaluated at various fixed values of x, as functions of time, and stored for subsequent comparisons. Figures 2 and 3 show the histories of u and w at $x = .25$mm. As is evident from Figure 3, the numerical calculation of w is not free from noise. Nevertheless, we use the term "exact solution" for any history obtained by the above numerical computation of the direct problem. All histories are records consisting of 400 equispaced samples on the time interval [0,100] milliseconds.

3. The Inverse Problem

The physical region of interest here is the x interval between 0 and .25mm. The histories in Figures 2 and 3 simulate what might have been recorded by a thermocouple at .25mm away from the inside wall as the shell is fired. The object is to use such data to reconstruct the temperature and gradient histories, arbitrarily close to the inside wall. In actuality, two thermocouple readings are necessary at $x = x_0$ and $x = x_1$, with $x_0 < .25 < x_1$; a well posed direct calculation, as in Section 2 above, then yields u and w at $x = .25$mm. As noted in the references given in [1], this type of inverse problem occurs in a variety of heat transfer contexts. The purpose of our computational experiment is as follows:

a) To demonstrate the feasibility of the inverse calculation in a realistic situation in which rapidly varying solutions

and nonlinearity play a role. As may be seen from Figure 1, the postulated temperature at the wall rises from 300° K to almost 1000° K in the first 10 milliseconds. In this temperature range, the conduction coefficient $a(u)$ given by (2.4) undergoes a 280 percent change.

b) To demonstrate the robustness of our algorithm with noisy data and a fine grid.

c) The regularized marching procedure we shall use is a powerful general method, applicable to other ill-posed evolutionary partial differential equations, linear and nonlinear. As used here, it is an adaptation to the nonlinear case of an algorithm which is rigorously justified in the constant coefficient case. While the heuristic "local mode analysis" underlying our regularization is likely to be valid in many other cases of ill-posed initial value problems, there is a need for well-documented realistic inverse calculations.

Let $z = \ell - x$ and let a_0, a_1 be positive constants such that

$$(3.1) \qquad 0 < a_0 < a(u) < a_1 \ .$$

Let $b(u) = \frac{da}{du}$, and let $v(z,t) = u(x,t)$. Using (2.5), we may write (2.1) as an equivalent first order system

$$(3.2) \qquad v_z = \frac{w}{a(v)} , \qquad w_z = v_t , \qquad 0 < z < \ell, \ t > 0,$$

with the subscript notation used for partial derivatives.

to be integrated in the direction of increasing z from $z = 0$ to $z = \ell$; we use the initial values given in Figures 2 and 3 and the following boundary conditions at $t = 0$,

$$v(z,0) = 300^\circ\ \text{K}, \quad w(z,0) = 0. \tag{3.3}$$

Let Δz be the increment in the z-variable and let $\ell = (N + 1)\Delta z$. Let $v^n(t)$, $w^n(t)$ denote, respectively, $v(n\Delta z,t)$, $w(n\Delta z,t)$, for $0 \leq n \leq N + 1$. The following finite difference approximation is second order accurate and explicit,

$$v^{n+1}(t) = v^n(t) + \frac{\Delta z\ w^n(t)}{a(v^n(t))} + \frac{\Delta z^2\ v_t^n(t)}{2a(v^n(t))} - \frac{\Delta z^2\ b(v^n(t))[w^n(t)]^2}{2a^3(v^n(t))} \tag{3.4}$$

$$w^{n+1}(t) = w^n(t) + \Delta z v_t^n(t) + \frac{\Delta z^2 w_t^n(t)}{2a(v^n(t))} - \frac{\Delta z^2\ b(v^n(t))\ w^n(t)\ v_t^n(t)}{2a^2(v^n(t))} \tag{3.5}$$

An effective way of implementing this scheme is to use cubic spline interpolation at the 400 equally spaced mesh points on the time interval $[0,T]$. Differentiating the spline function produces $O(\Delta t^3)$ accurate derivatives $v_t^n(t)$, $w_t^n(t)$ at these same mesh points, and hence $v^{n+1}(t)$, $w^{n+1}(t)$ from (3.4), (3.5). The next step is to stabilize this process by filtering each of these functions in the

frequency domain. This is accomplished by dividing the k^{th} Fourier coefficient by the precomputed weight λ_k, where

$$(3.6) \qquad \lambda_k = \left(1 + \omega^2 \exp\left[\,\ell\ \frac{2|k|\pi}{a_0 T}\,\right]\right)^{\Delta z/\ell} .$$

there $\omega = (\frac{\varepsilon}{M})$ is the L^2 noise to signal ratio. See [1].

With ℓ = .25mm, the x-interval $[0,\ell]$ was divided into 450 equally spaced mesh points, and the above procedure was implemented with ω = .001. Figures 4 through 11 summarize the comparison between exact and computed solutions at the <u>interior</u> location x = .056mm. An idea of the relative errors in the calculation is easily gained from Figures 7 and 11. Although the "logarithmic convexity" estimates in Theorem 1 of [1] degenerate at the wall, the computation was pursued for 450 cycles and approximations to the temperature and gradient histories at the wall were obtained. These are shown in Figures 13 and 17. The "exact" temperature and gradient histories at the wall are shown in Figures 12 and 16. Clearly, slight inaccuracies in the well-posed direct calculation of u(x,t) near x = 0, lead to a rather noisy determination of the exact w(x,t) at x = 0; in particular, the pronounced spike near t = 40 milliseconds in Figure 16 is a numerical artifact which should be disregarded. Nonetheless, we have chosen to compare the computed gradient history in Figure 17 with the wall profile given in Figure 16. As is evident from Figures 15 and 19, the wall estimates obtained by solving the inverse problem are quite reliable. This is especially true during the first twenty or so milliseconds where peak values are achieved.

4. Conclusions

A regularized marching algorithm has been shown to be effective in solving nonlinear inverse heat transfer problems in gun barrels. In [2], a similar technique was used successfully on linear backwards parabolic equations with highly variable coefficients. More recently, success has also been achieved on other unstable examples involving Burgers' equation with the time direction reversed.

Future work should be directed towards problems in two or more space dimensions in general domains, in the context of heat transfer and fluid mechanics.

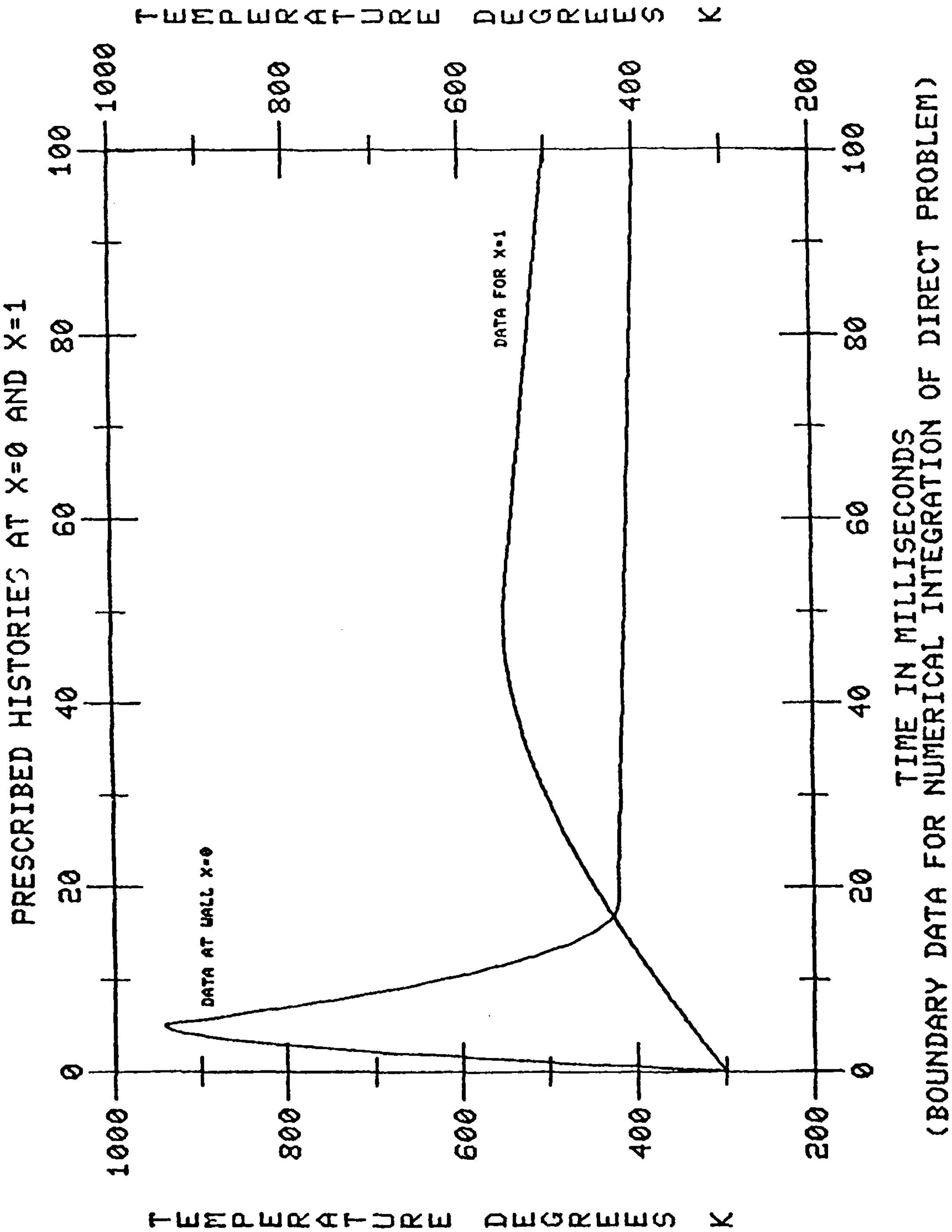

PRESCRIBED HISTORIES AT X=0 AND X=1
TEMPERATURE DEGREES K
1000
800
600
400
200
0
20
40
60
80
100
DATA AT WALL X=0
DATA FOR X=1
TEMPERATURE DEGREES K
TIME IN MILLISECONDS
(BOUNDARY DATA FOR NUMERICAL INTEGRATION OF DIRECT PROBLEM)

FIGURE 1

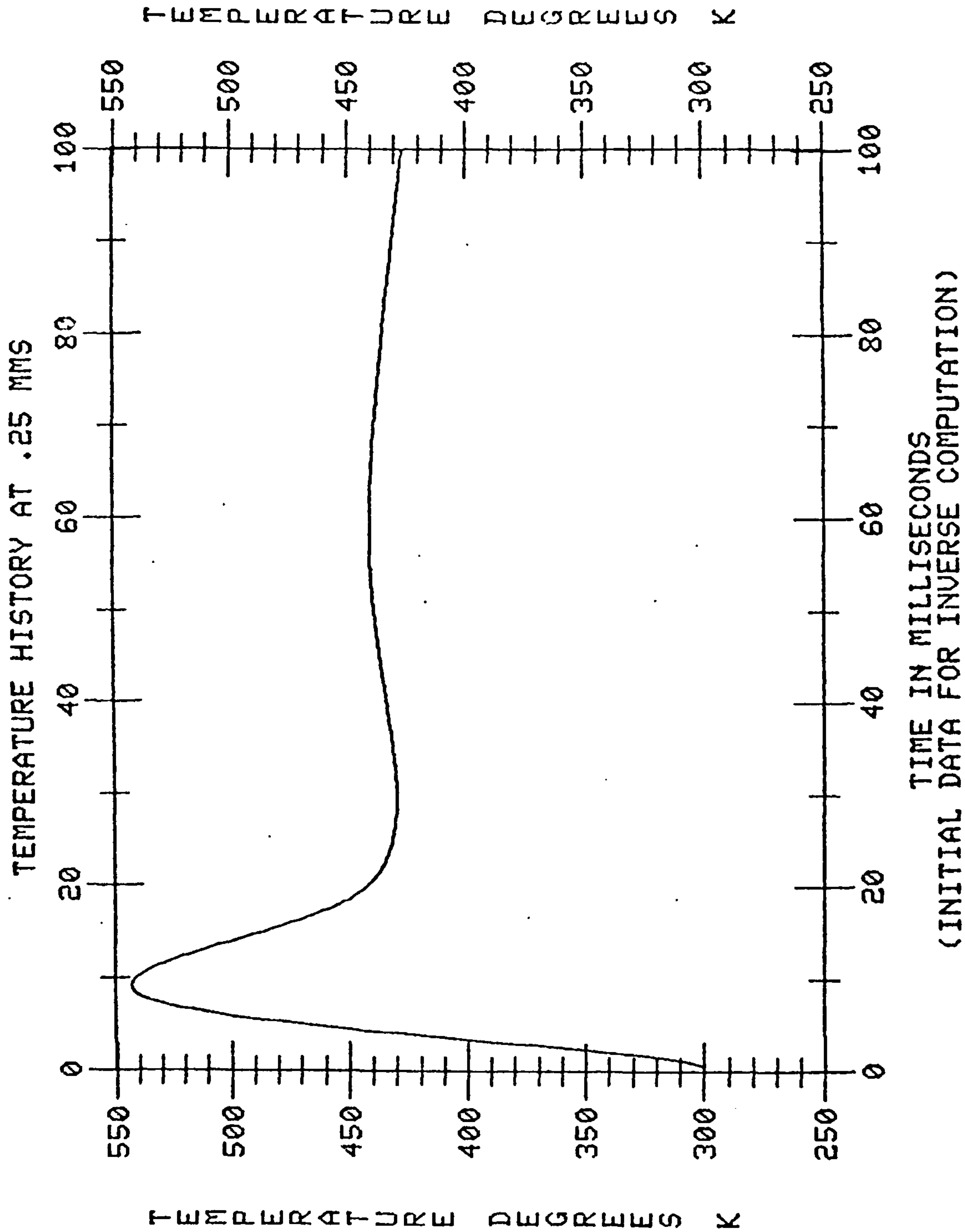

TEMPERATURE HISTORY AT .25 MMS
TIME IN MILLISECONDS
(INITIAL DATA FOR INVERSE COMPUTATION)
TEMPERATURE DEGREES K
0
20
40
60
80
100
250
300
350
400
450
500
550

FIGURE 2

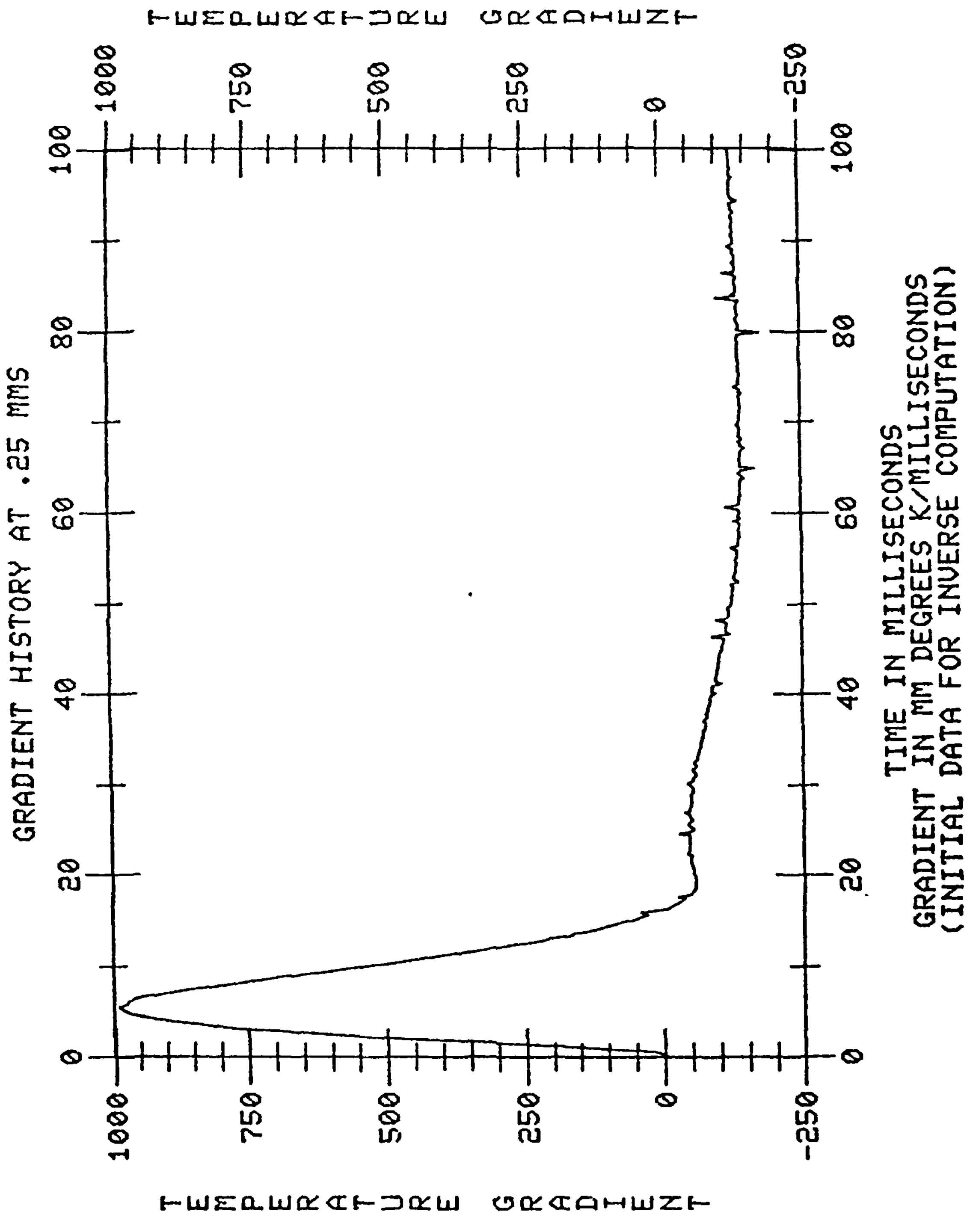

GRADIENT HISTORY AT .25 MMS
TEMPERATURE GRADIENT
1000
750
500
250
0
-250
0
20
40
60
80
100
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
(INITIAL DATA FOR INVERSE COMPUTATION)

FIGURE 3

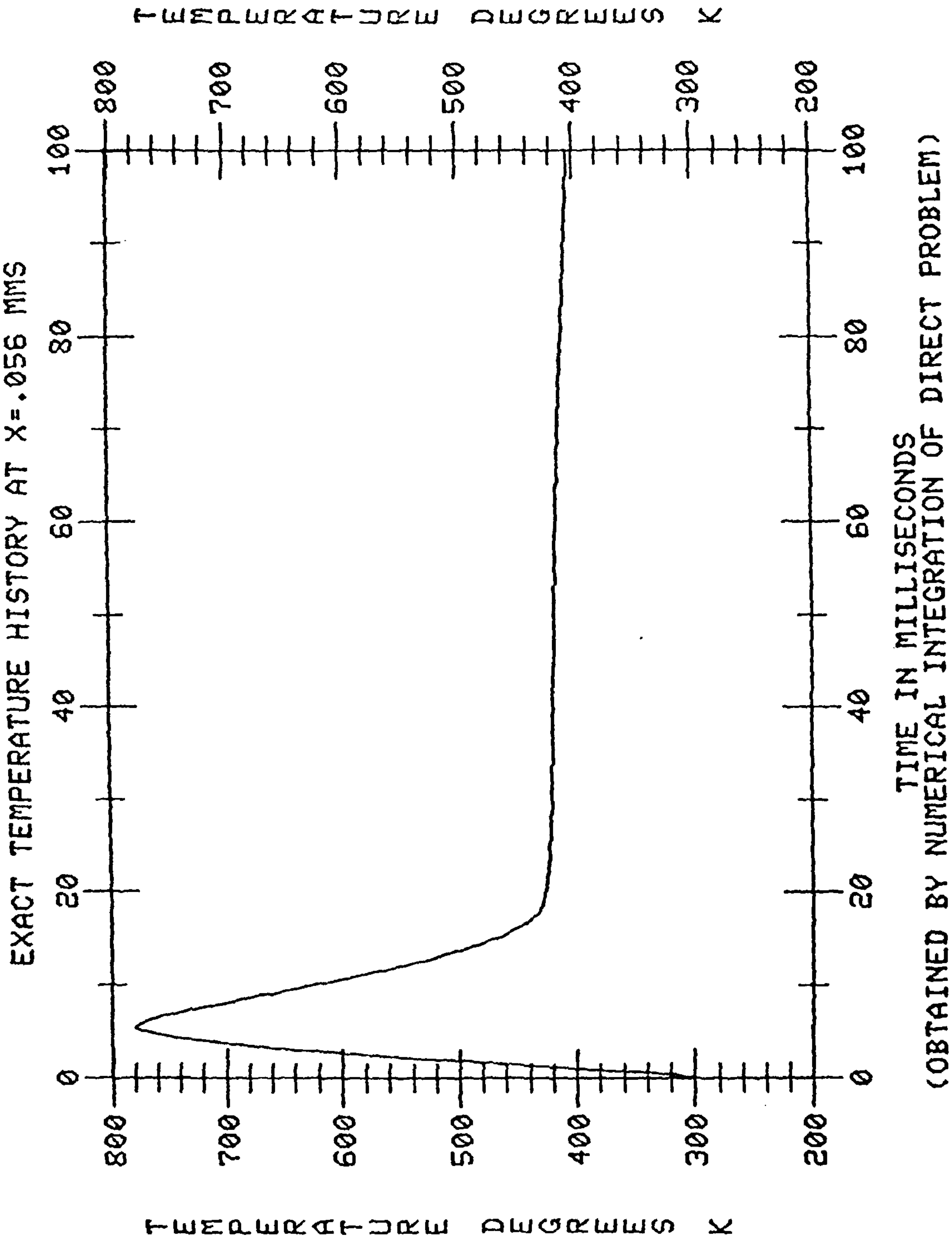

EXACT TEMPERATURE HISTORY AT X=.056 MMS
TIME IN MILLISECONDS
(OBTAINED BY NUMERICAL INTEGRATION OF DIRECT PROBLEM)
0
20
40
60
80
100
800
700
600
500
400
300
200

FIGURE 4

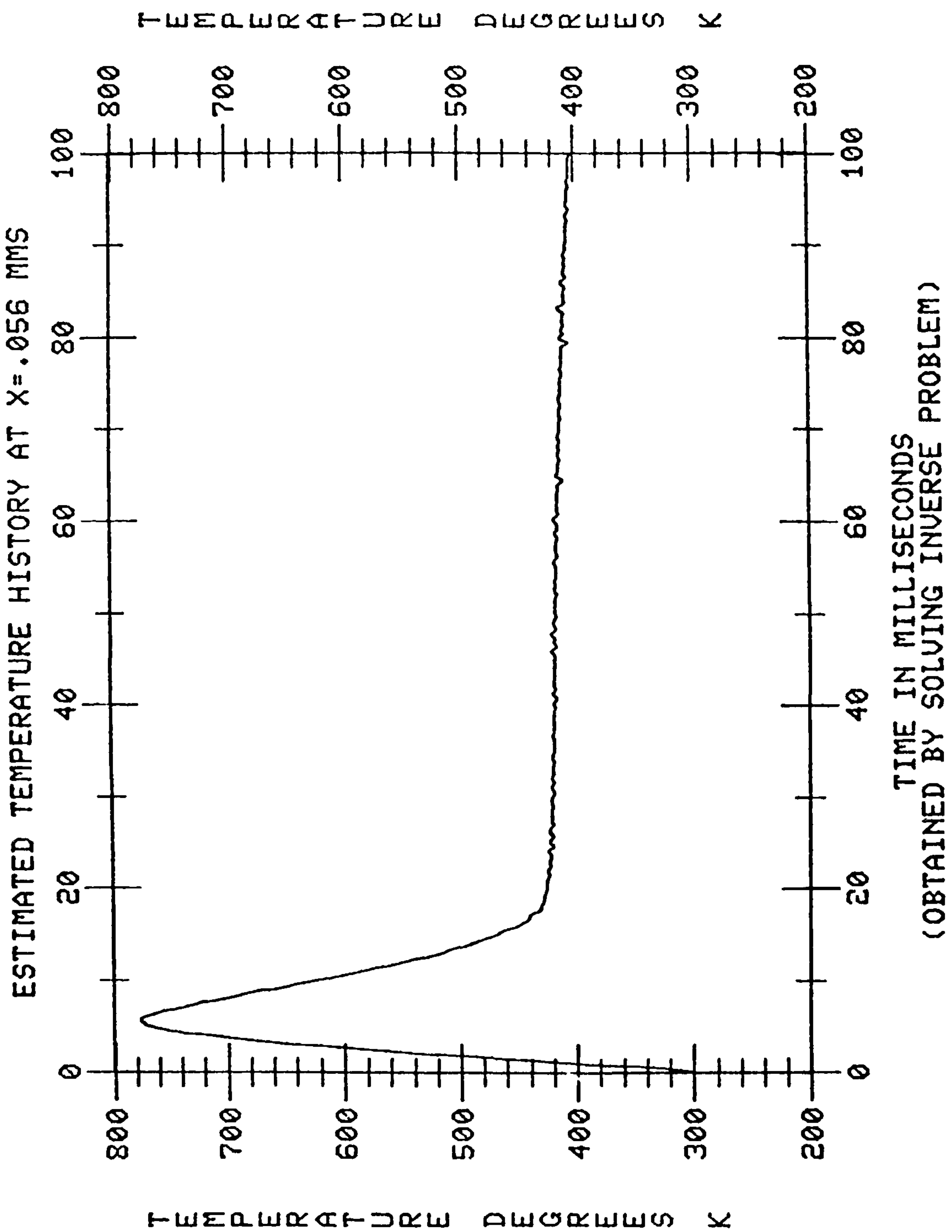
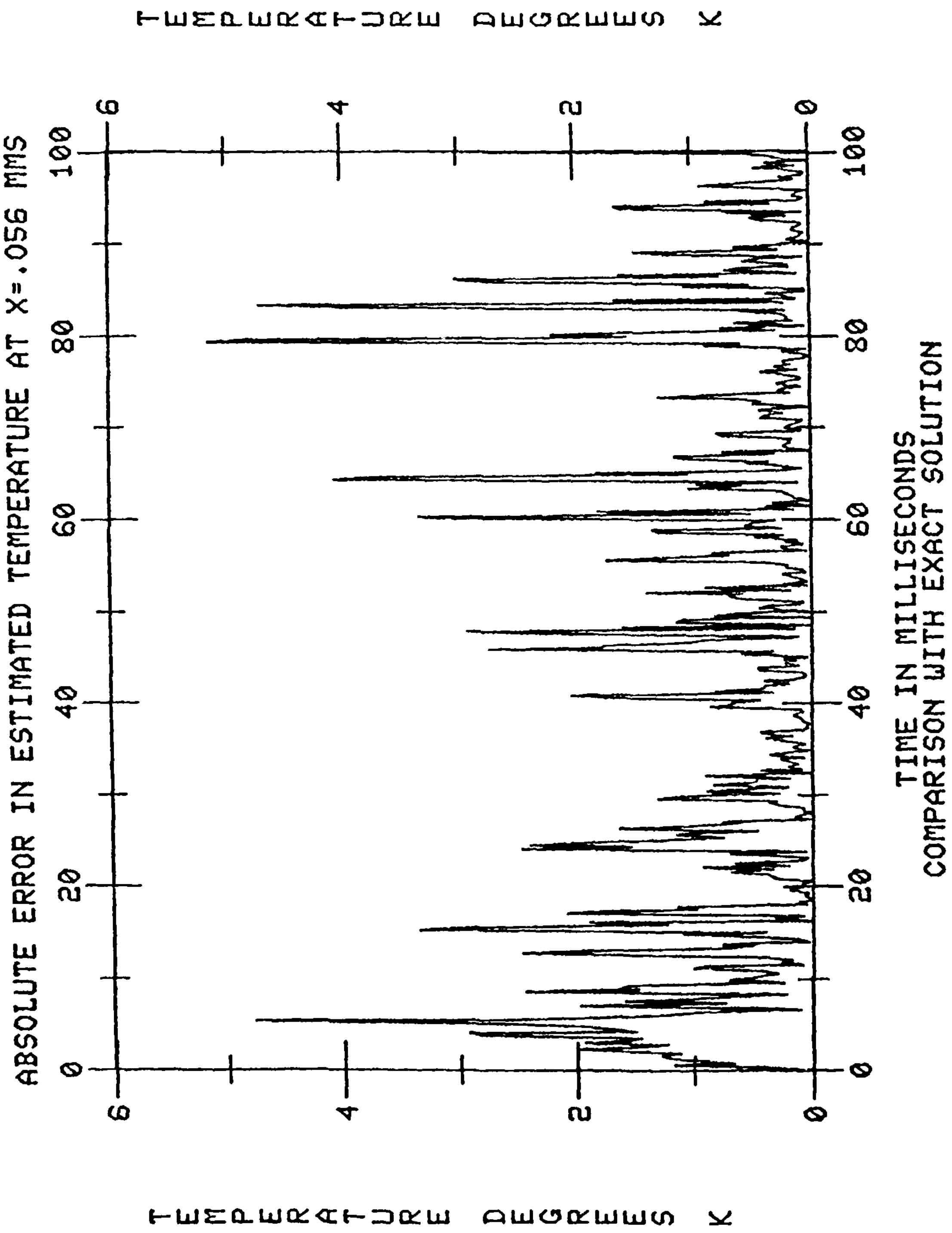

ESTIMATED TEMPERATURE HISTORY AT X=.056 MMS
TIME IN MILLISECONDS
(OBTAINED BY SOLVING INVERSE PROBLEM)
0
20
40
60
80
100
800
700
600
500
400
300
200

FIGURE 5



ABSOLUTE ERROR IN ESTIMATED TEMPERATURE AT X=.056 MMS
TEMPERATURE DEGREES K
0
2
4
6
0
20
40
60
80
100
TIME IN MILLISECONDS
COMPARISON WITH EXACT SOLUTION

FIGURE 6

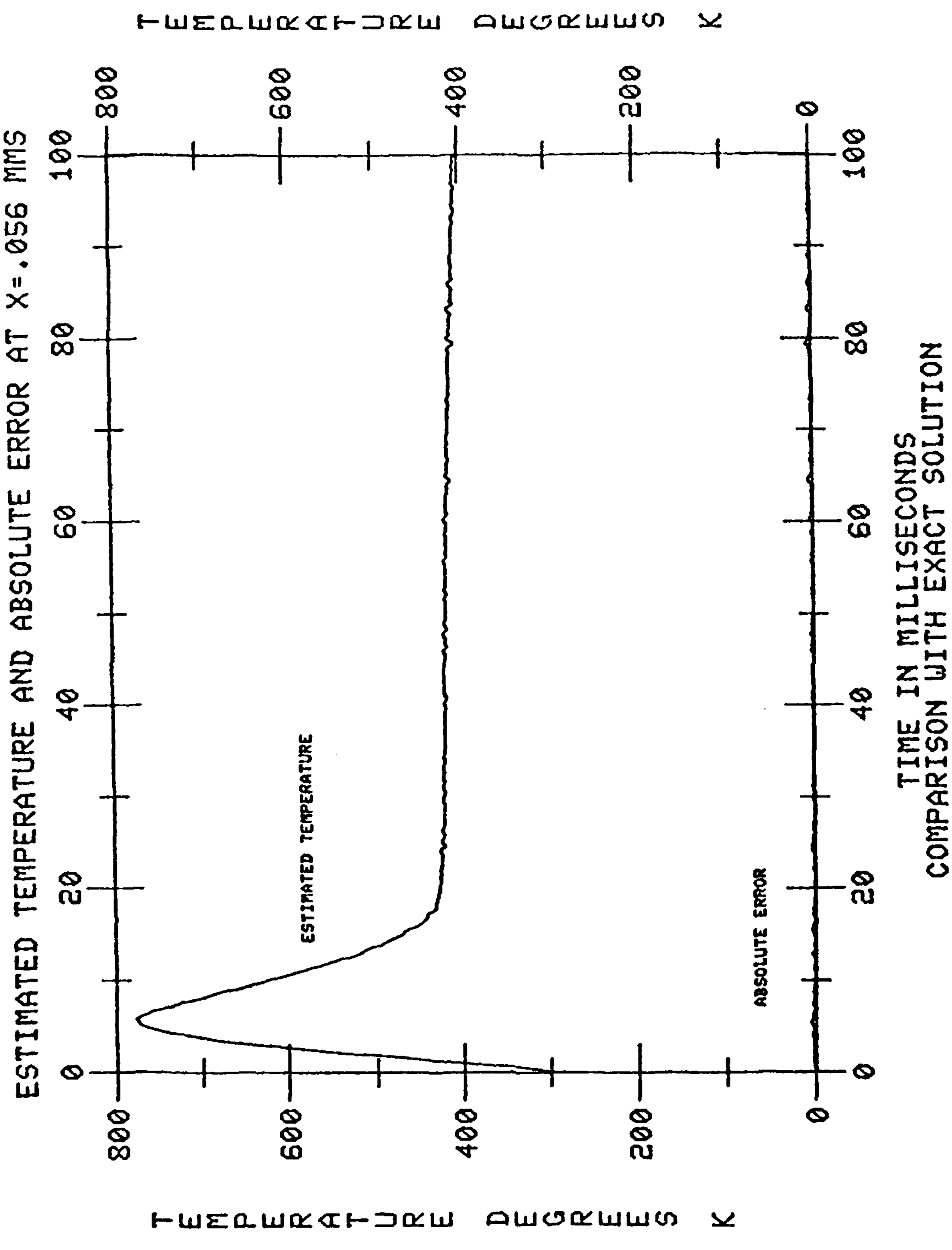

ESTIMATED TEMPERATURE AND ABSOLUTE ERROR AT X=.056 MMS
TIME IN MILLISECONDS
COMPARISON WITH EXACT SOLUTION
ESTIMATED TEMPERATURE
ABSOLUTE ERROR
TEMPERATURE DEGREES K
0 20 40 60 80 100
800 600 400 200 0

FIGURE 7

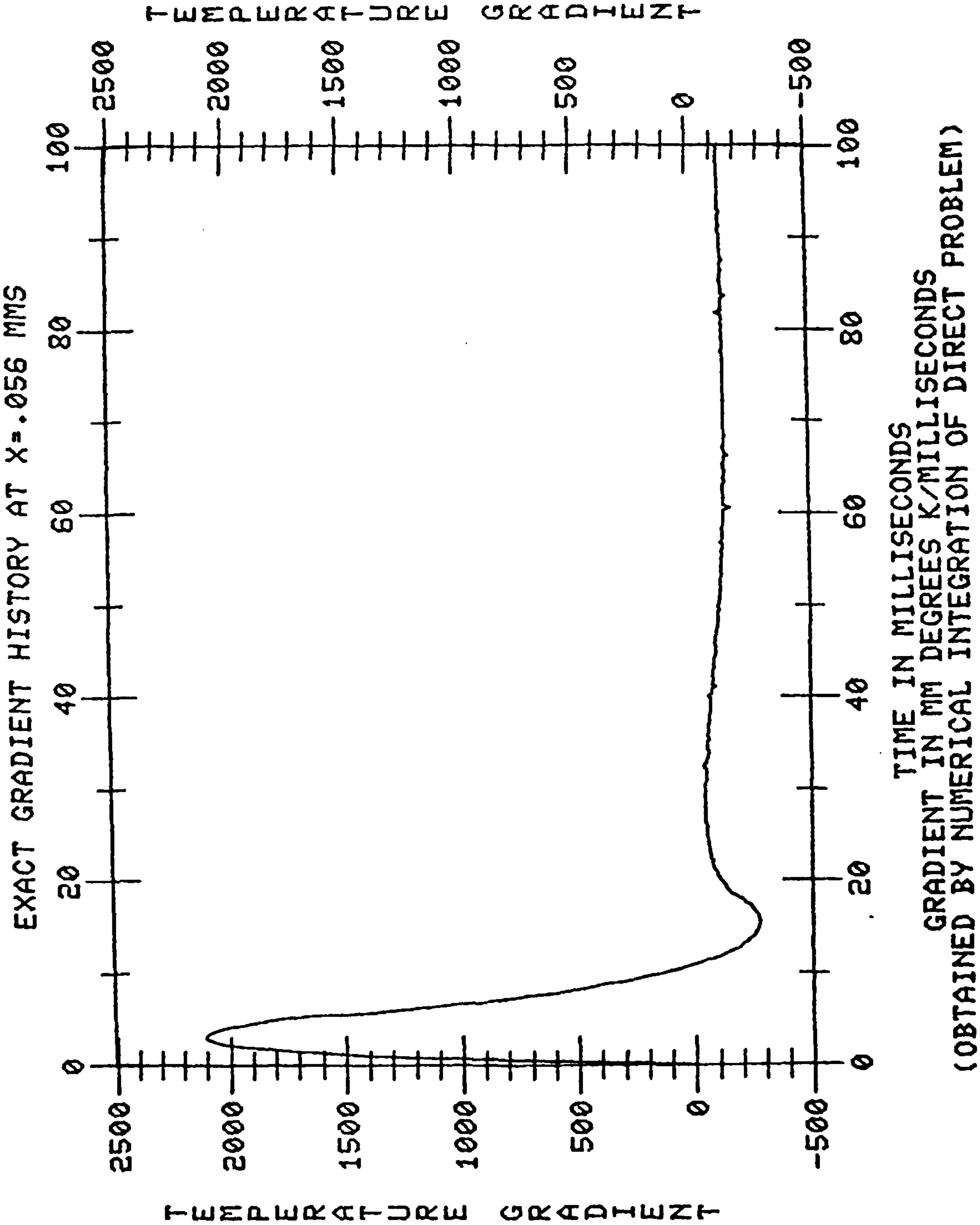

EXACT GRADIENT HISTORY AT X=.056 MMS
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
(OBTAINED BY NUMERICAL INTEGRATION OF DIRECT PROBLEM)
TEMPERATURE GRADIENT
2500
2000
1500
1000
500
0
-500
0
20
40
60
80
100
TEMPERATURE GRADIENT

FIGURE 8

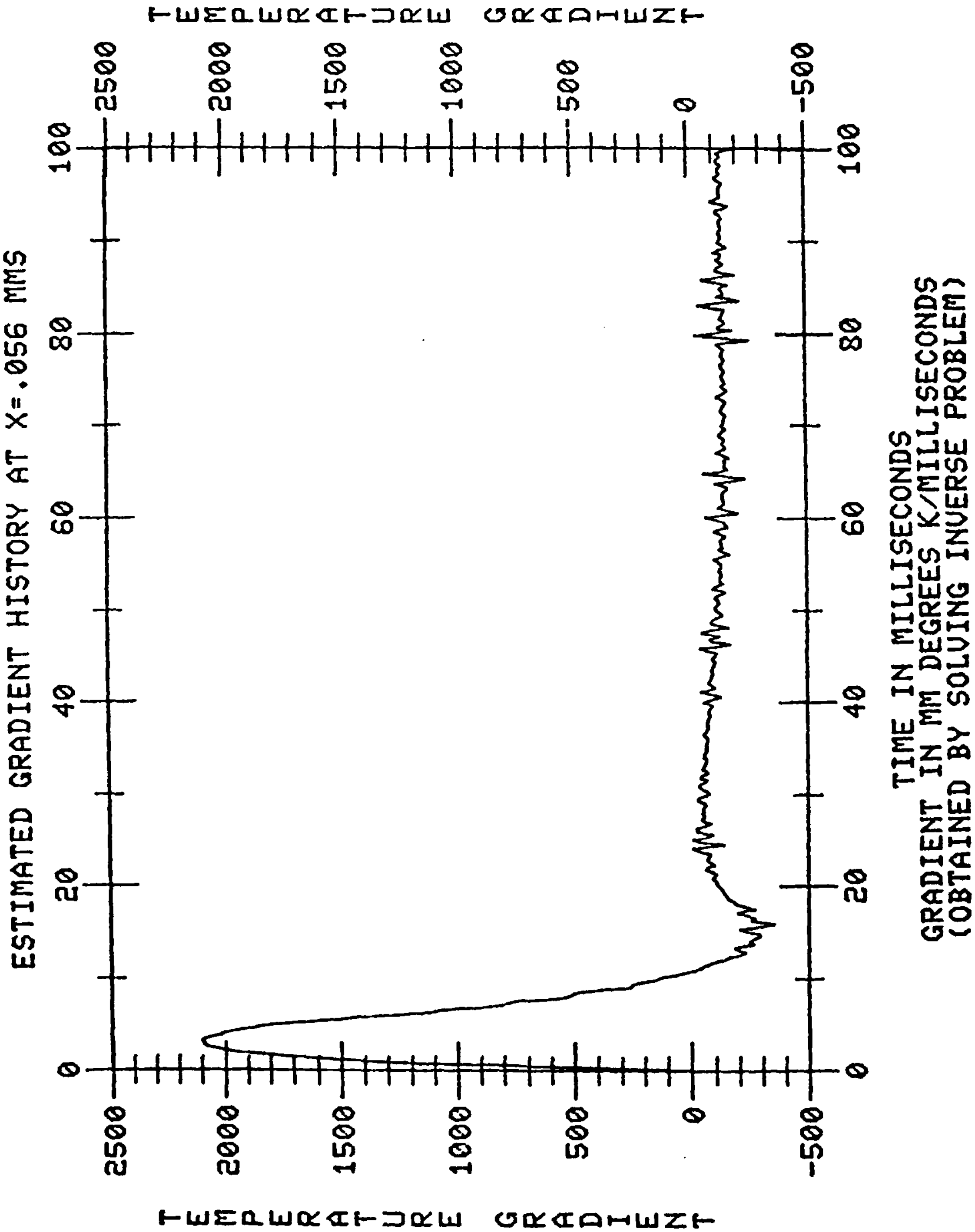

ESTIMATED GRADIENT HISTORY AT X=.056 MMS
TEMPERATURE GRADIENT
2500
2000
1500
1000
500
0
-500
0
20
40
60
80
100
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
(OBTAINED BY SOLVING INVERSE PROBLEM)

FIGURE 9

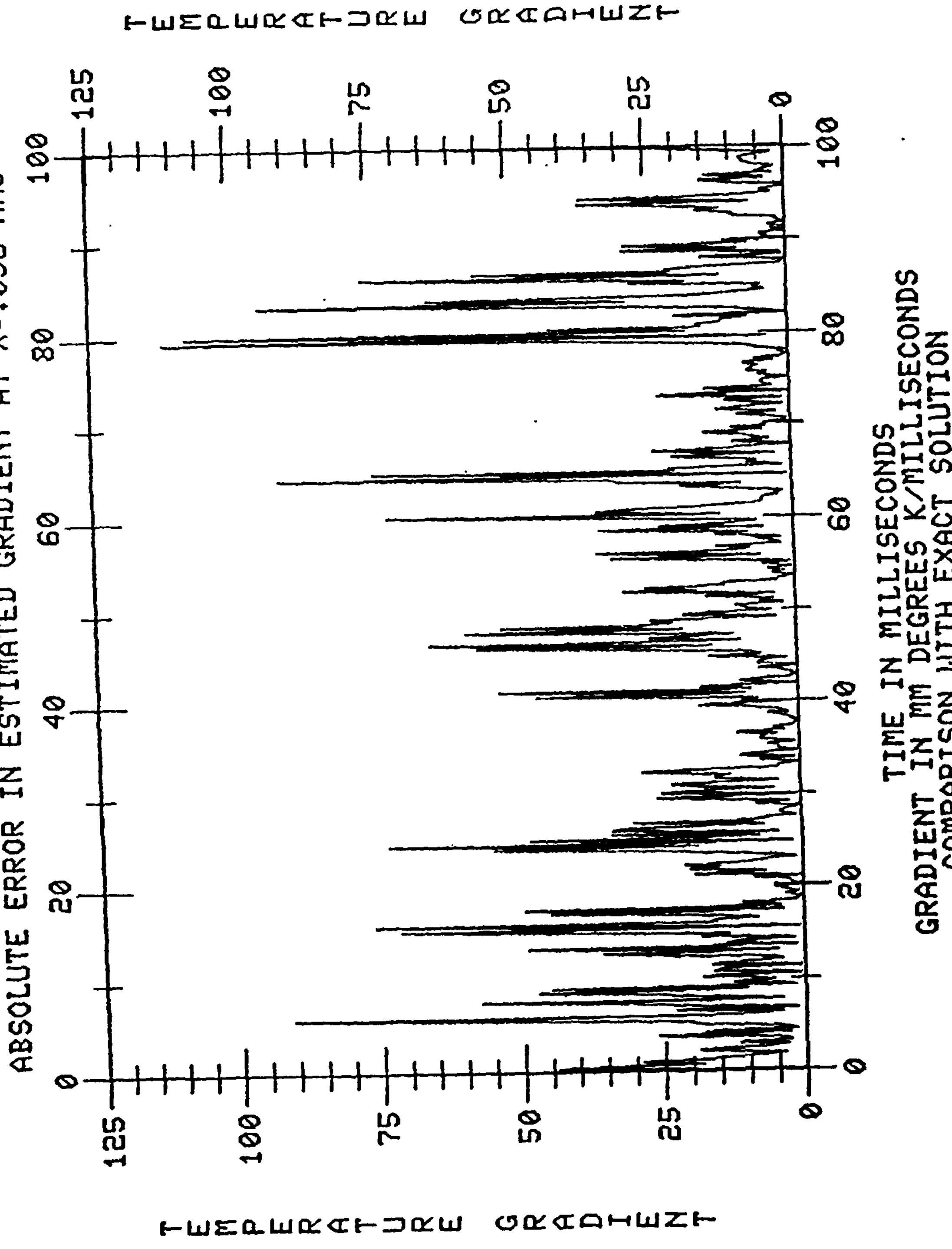

ABSOLUTE ERROR IN ESTIMATED GRADIENT AT X=.056 MMS
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
COMPARISON WITH EXACT SOLUTION
TEMPERATURE GRADIENT
TEMPERATURE GRADIENT
0
20
40
60
80
100
125
100
75
50
25
0

FIGURE 10

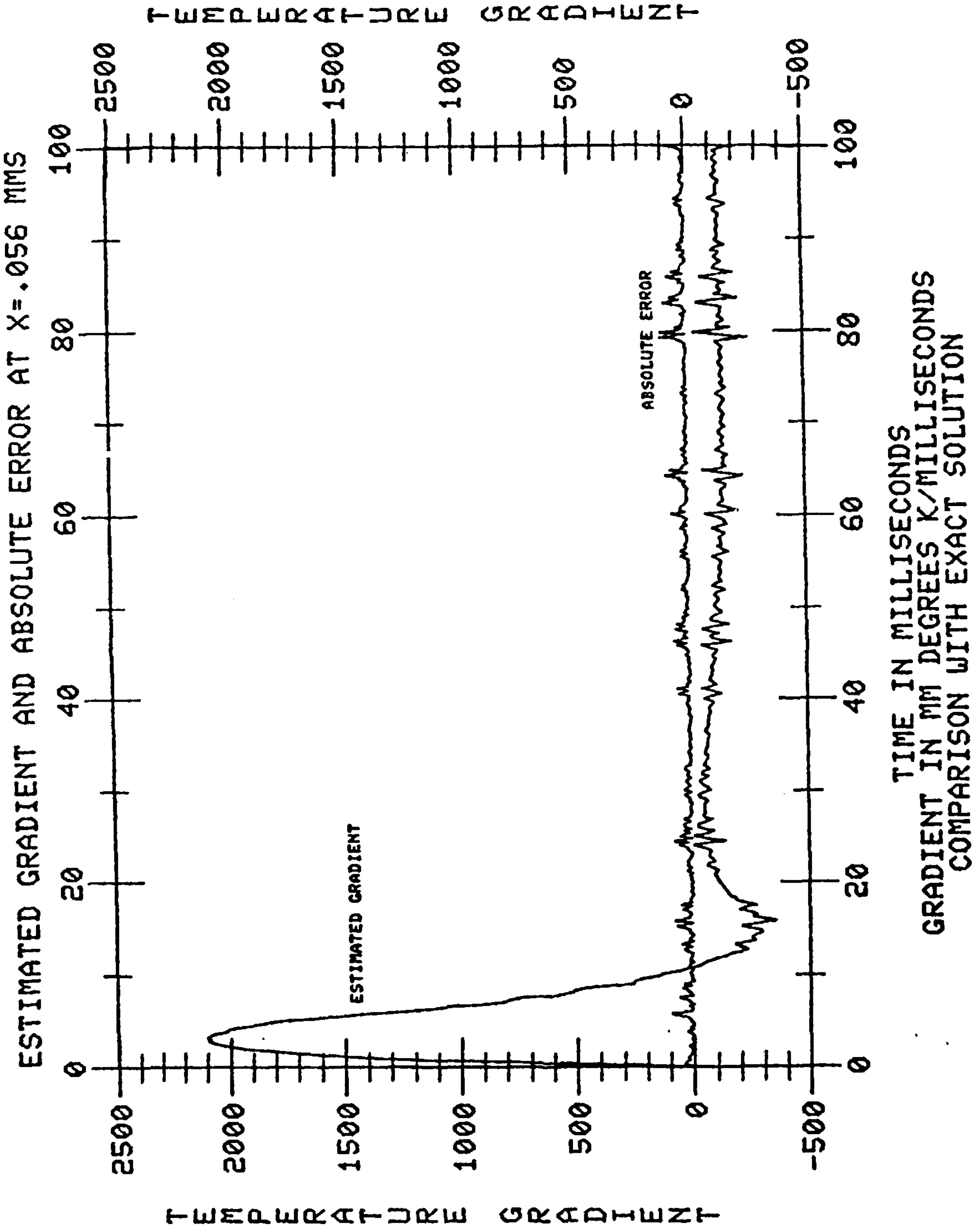

ESTIMATED GRADIENT AND ABSOLUTE ERROR AT X=.056 MMS
TEMPERATURE GRADIENT
2500
2000
1500
1000
500
0
-500
0
20
40
60
80
100
ESTIMATED GRADIENT
ABSOLUTE ERROR
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
COMPARISON WITH EXACT SOLUTION

FIGURE 11

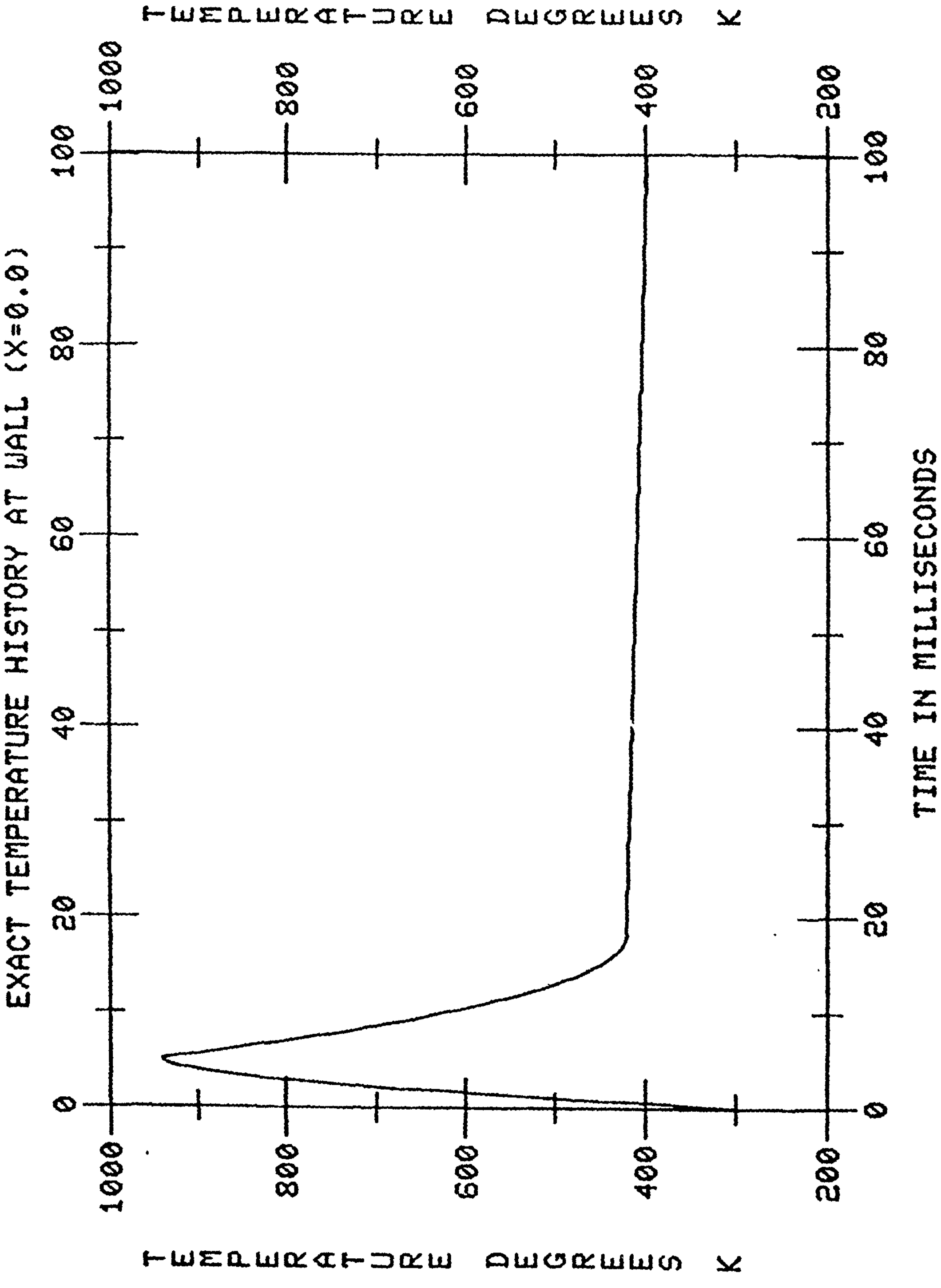

EXACT TEMPERATURE HISTORY AT WALL (X=0.0)
TIME IN MILLISECONDS
TEMPERATURE DEGREES K
TEMPERATURE DEGREES K
0
20
40
60
80
100
1000
800
600
400
200

FIGURE 12

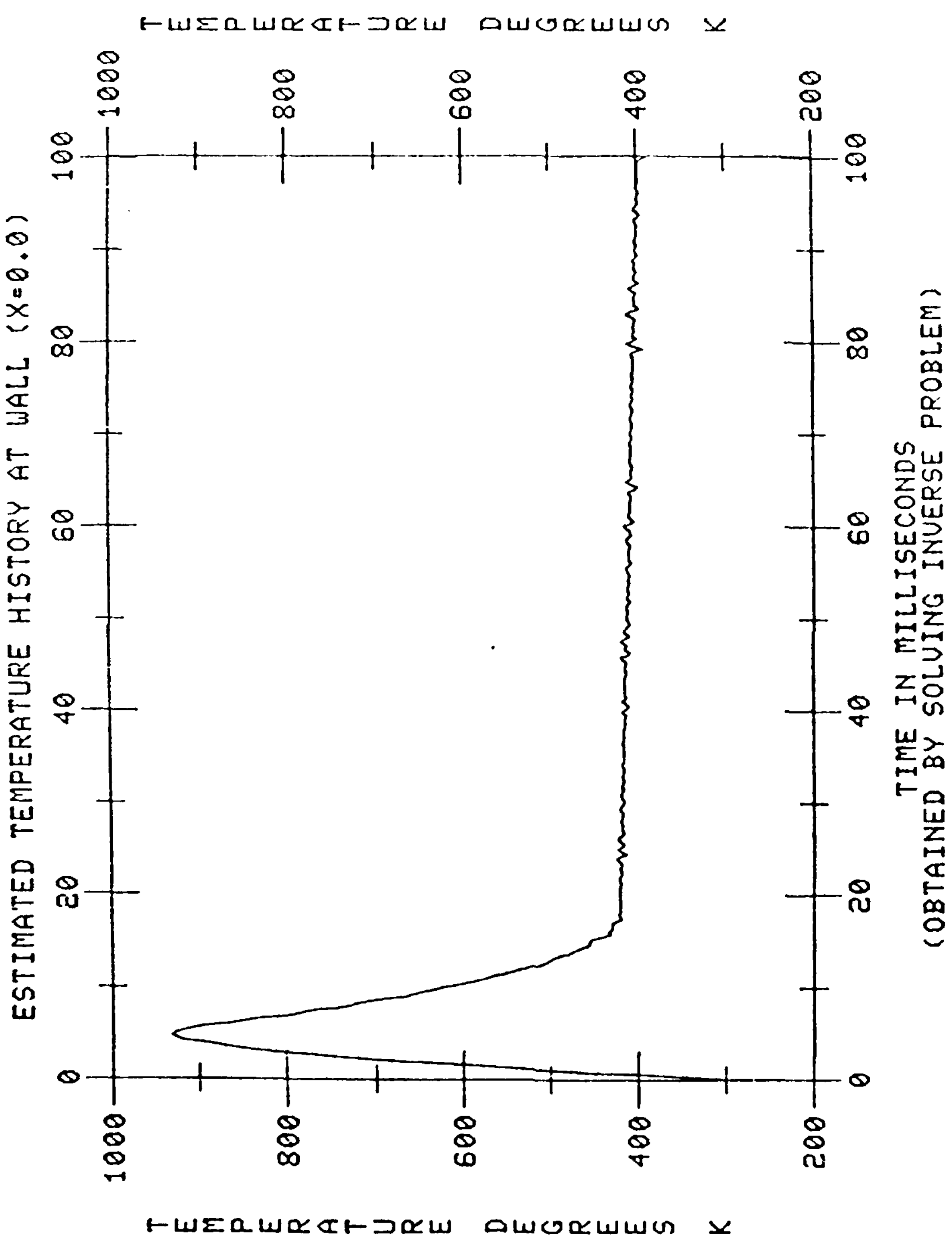

ESTIMATED TEMPERATURE HISTORY AT WALL (X=0.0)
TIME IN MILLISECONDS
(OBTAINED BY SOLVING INVERSE PROBLEM)
TEMPERATURE DEGREES K
0
20
40
60
80
100
200
400
600
800
1000

FIGURE 13

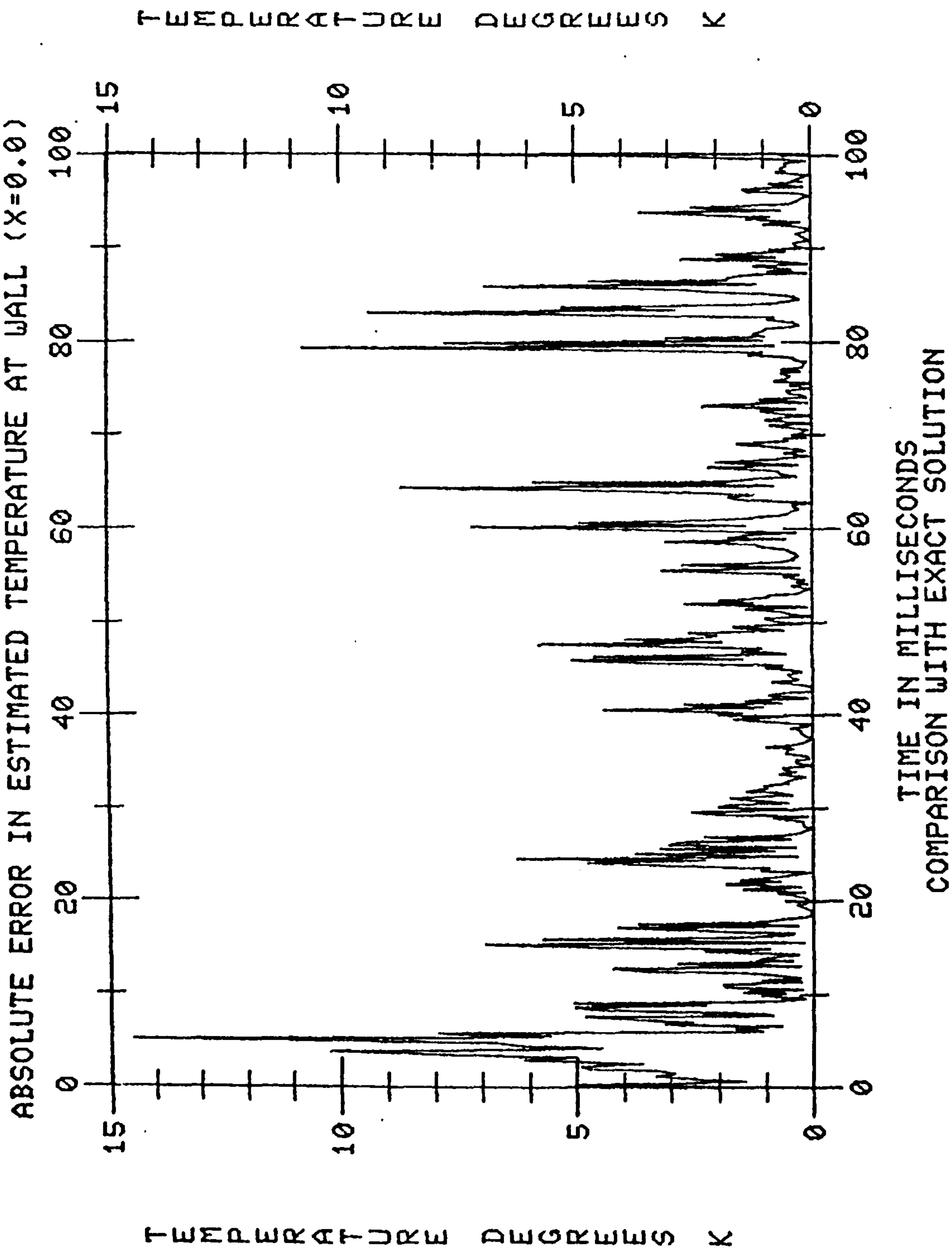

ABSOLUTE ERROR IN ESTIMATED TEMPERATURE AT WALL (X=0.0)
TIME IN MILLISECONDS
COMPARISON WITH EXACT SOLUTION
0
20
40
60
80
100
0
5
10
15

FIGURE 14

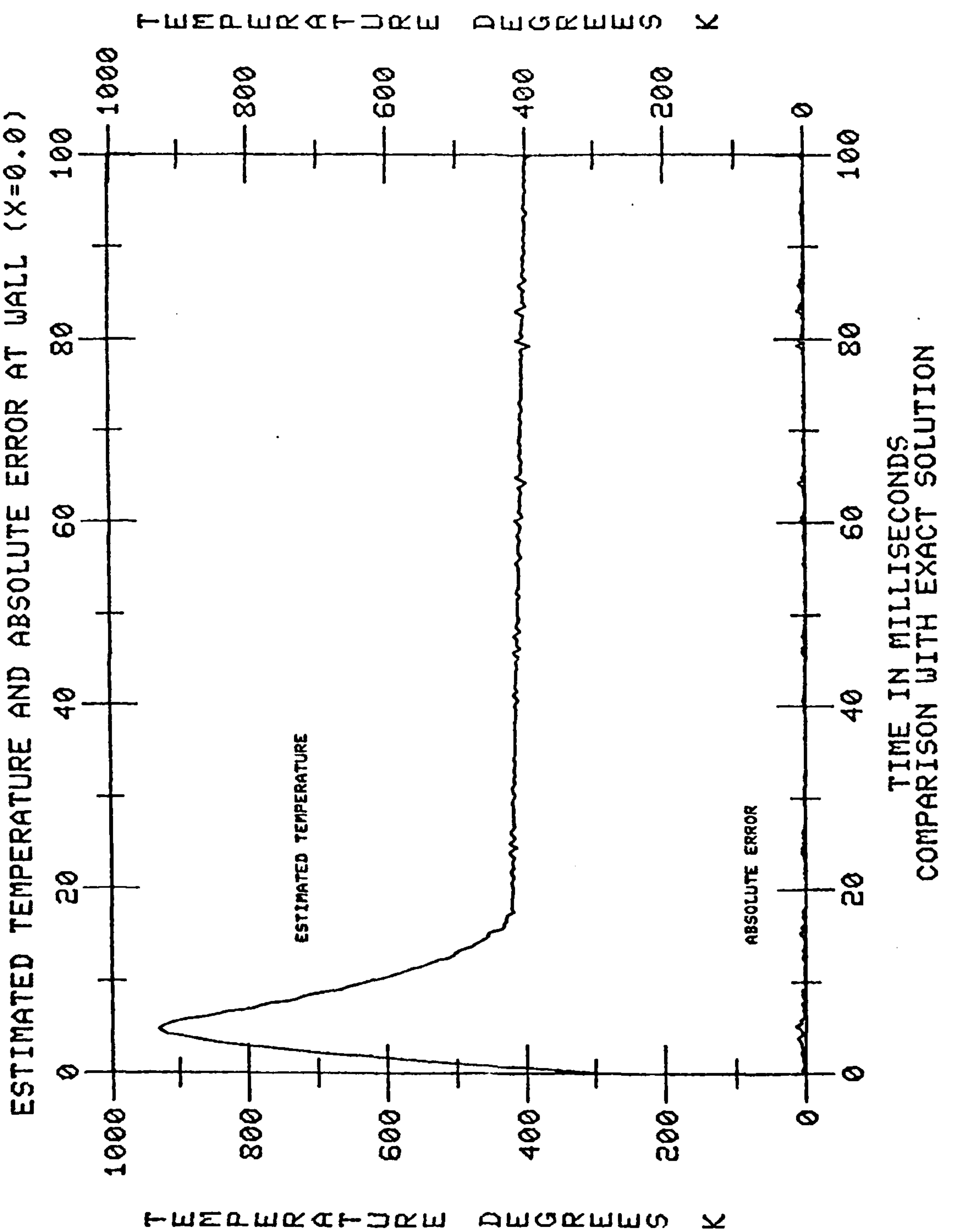

ESTIMATED TEMPERATURE AND ABSOLUTE ERROR AT WALL (X=0.0)
TEMPERATURE DEGREES K
1000
800
600
400
200
0
0
20
40
60
80
100
ESTIMATED TEMPERATURE
ABSOLUTE ERROR
TIME IN MILLISECONDS
COMPARISON WITH EXACT SOLUTION

FIGURE 15

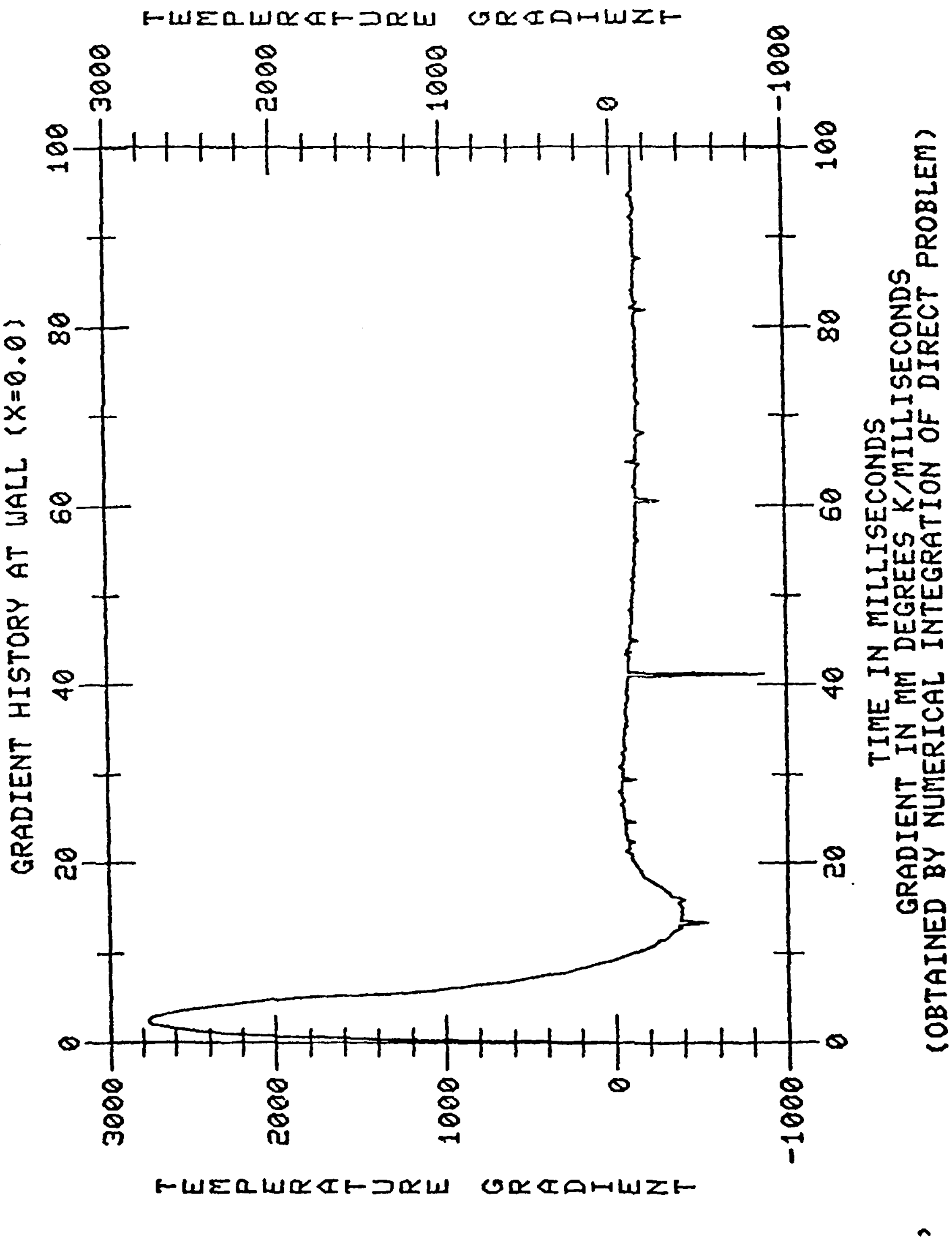

GRADIENT HISTORY AT WALL (X=0.0)
TEMPERATURE GRADIENT
3000
2000
1000
0
-1000
0
20
40
60
80
100
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
(OBTAINED BY NUMERICAL INTEGRATION OF DIRECT PROBLEM)

FIGURE 16

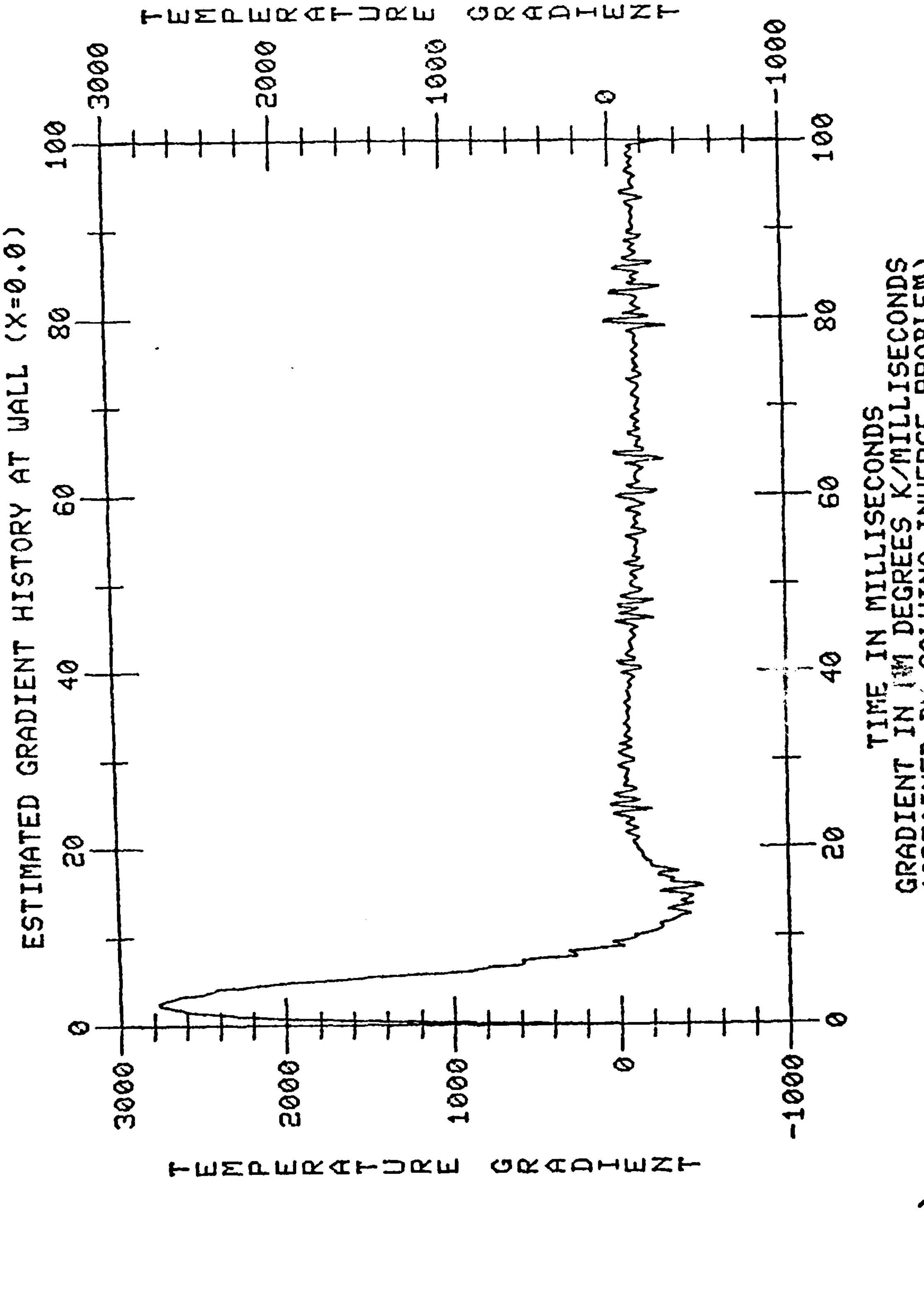

ESTIMATED GRADIENT HISTORY AT WALL (X=0.0)
TEMPERATURE GRADIENT
3000
2000
1000
0
-1000
0
20
40
60
80
100
TIME IN MILLISECONDS
GRADIENT IN [illegible]M DEGREES K/MILLISECONDS
(OBTAINED BY SOLVING INVERSE PROBLEM)

FIGURE 17

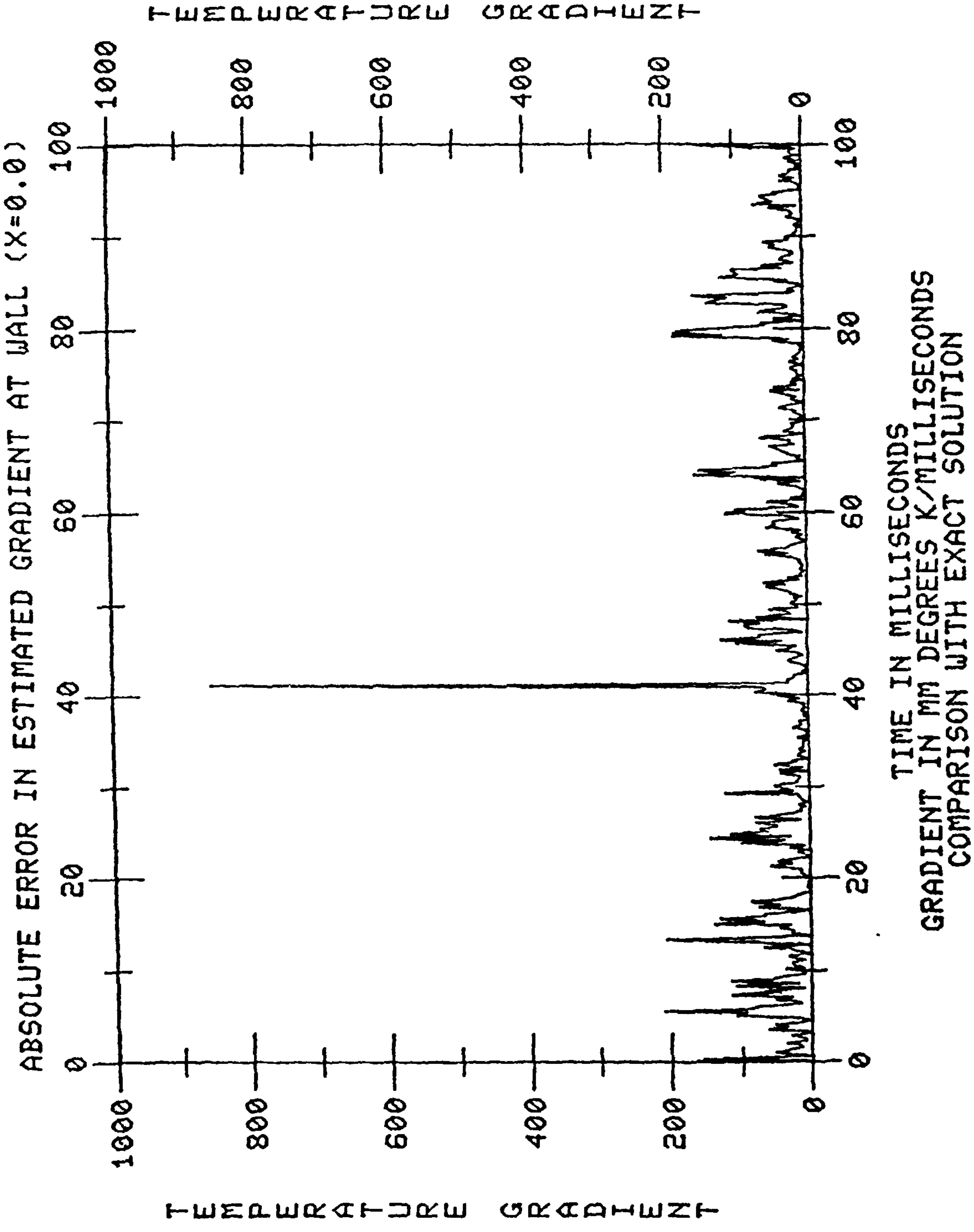

ABSOLUTE ERROR IN ESTIMATED GRADIENT AT WALL (X=0.0)
TEMPERATURE GRADIENT
0
200
400
600
800
1000
0
20
40
60
80
100
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
COMPARISON WITH EXACT SOLUTION

FIGURE 18

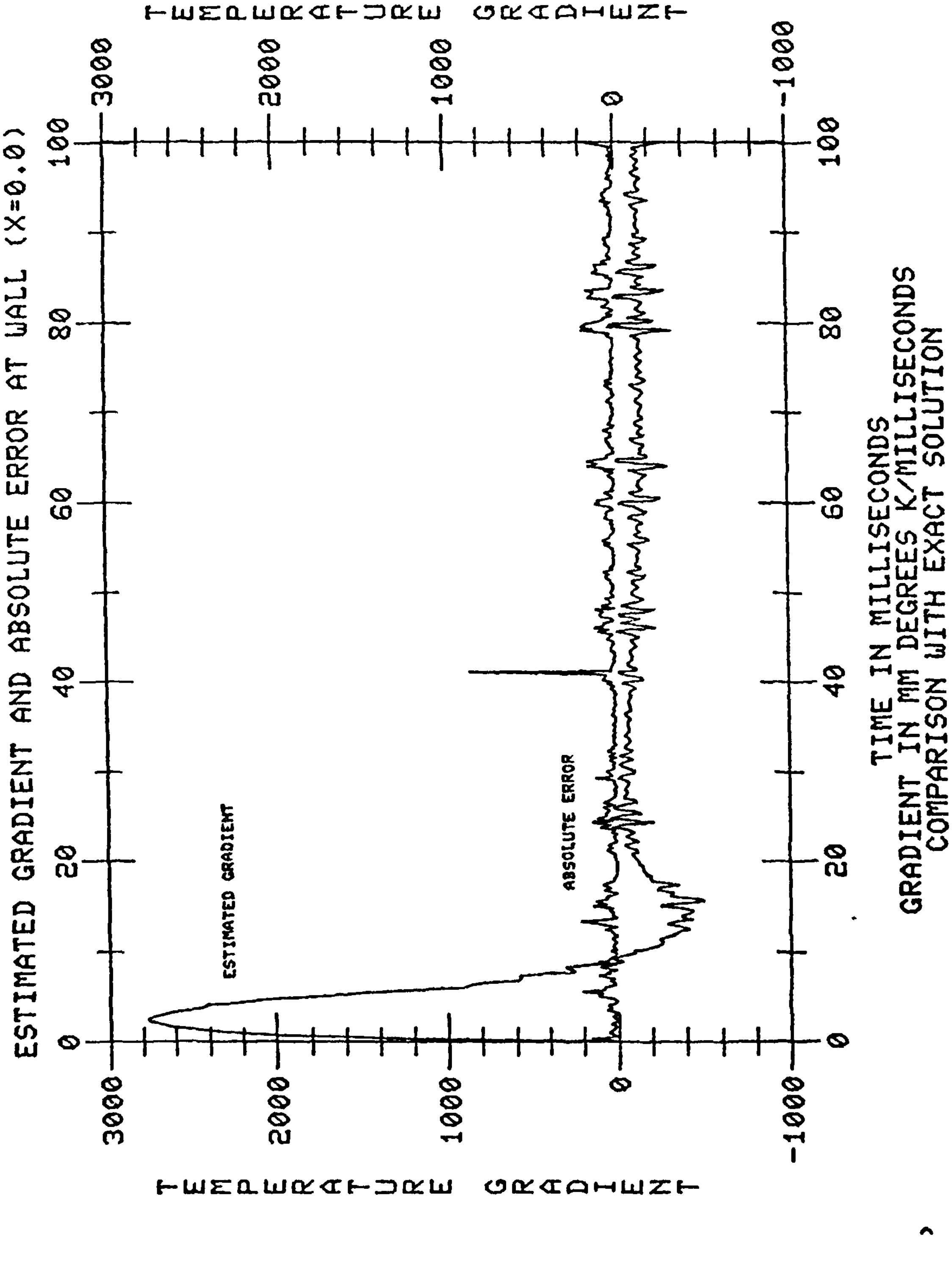

ESTIMATED GRADIENT AND ABSOLUTE ERROR AT WALL (X=0.0)
ESTIMATED GRADIENT
ABSOLUTE ERROR
TIME IN MILLISECONDS
GRADIENT IN MM DEGREES K/MILLISECONDS
COMPARISON WITH EXACT SOLUTION
TEMPERATURE GRADIENT
3000
2000
1000
0
-1000
0
20
40
60
80
100

FIGURE 19

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